UNIT III

Electromagnetic waves in conductors

Electric and magnetic fields satisfy identical eq's) in a medium in which there is no free charge

$$\nabla^2 E - \varepsilon\mu \frac{\partial^2 E}{\partial t^2} - \sigma\mu \frac{\partial E}{\partial t} = 0 \quad \text{--- (1)}$$

and $$\nabla^2 H - \varepsilon\mu \frac{\partial^2 H}{\partial t^2} - \sigma\mu \frac{\partial H}{\partial t} = 0 \quad \text{--- (2)}$$

Both these equations contain damping terms proportional to the conductivity of the medium.

We now find the plane wave solution of Maxwell's eq's for a conducting medium. ~~...~~ Let's assume that the field vector E and H vary harmonically with time, i.e.

$$E(r,t) = E_0 e^{i(k\cdot r - \omega t)} \quad \text{--- (3)}$$

$$H(r,t) = H_0 e^{i(k\cdot r - \omega t)} \quad \text{--- (4)}$$

Substituting eq^n (3) in eq^n (1)

$$-k^2 E(r,t) + \varepsilon\mu\omega^2 E(r,t) + i\sigma\mu\omega E(r,t) = 0$$

i.e. $$\left[k^2 - \varepsilon\mu\omega^2 - i\sigma\mu\omega\right] E(r,t) = 0 \quad \text{--- (5)}$$

$$\Rightarrow \quad k^2 = \varepsilon\mu\omega^2\left[1 + \frac{i\sigma}{\varepsilon\omega}\right] \quad \text{--- (6)}$$

The first term corresponds to the displacement current and the second to the conduction current.

For any wave there is a functional relationship between the wave-number k and wave frequency ω. This relation is known as "dispersion relation".

For e.m. waves in vacuum, the dispersion relation is

$$k = \frac{\omega}{v}$$

for a conducting medium it is eq^n (6).

if $\sigma = 0$ (i.e. free space) $k^2 = \varepsilon\mu\omega^2 = \frac{\omega^2}{v^2}$

This relation will provide us with information regarding the nature of the propagation of electromagnetic waves inside a medium.

In a conducting medium, the propagation vector k is complex.

We may express it as

$$k = \alpha + i\beta \quad \text{--- (7)}$$

Squaring and comparing with eq^n (6)

$$\alpha^2 - \beta^2 = \varepsilon\mu\omega^2 \quad \text{--- (8)}$$

$$2\alpha\beta = \sigma\mu\omega$$

Solving these eq^n's

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[1 + \left\{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{1/2}\right]^{\frac{1}{2}} \quad \text{--- (9)}$$

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[-1 + \left\{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2\right\}^{1/2}\right]^{1/2} \quad \text{--- (10)}$$

We have to take possible square root in order to yield the proper form for k in free space

Since $k = \alpha + i\beta$ eq^n (3) and (4) can be written as

$$E = E_0 e^{-\beta r} e^{i(\alpha r - \omega t)}$$

$$H = H_0 e^{-\beta r} e^{i(\alpha r - \omega t)}$$

Above eq^n indicate that a plane wave can not propagate in a conducting medium without attenuation. When a plane wave is propagating in a conducting medium, the oscillating electric field in the wave setsup currents. Work must be done to drive the currents and some of the energy is dissipated as heat in the medium. This results into the ~~[illegible]~~ attenuation of the wave.

The quantity $\beta \rightarrow$ absorption coefficient and is a measure of this attenuation.

Behaviour of the medium at high and low frequencies?

as $\omega \rightarrow \infty$, $\beta \rightarrow 0$

$\Rightarrow$ At very high frequencies the e.m. waves will be able to travel through the medium.

At low frequencies $\beta \rightarrow$ will be finite and there will be appreciable damping of the wave.

The Skin effect: In eq^n(6) the term $i\sigma\mu\omega$ arises from the term involving $\frac{\partial E}{\partial t}$ in eq^n(2), i.e. from the conduction current, while the term $\varepsilon\mu\omega^2$ arises from the term involving $\frac{\partial^2 E}{\partial t^2}$ in the same eq^n, i.e. from the displacement current.

In all the conducting media, the conduction current dominates the displacement current, hence, to approximation we neglect the middle term in eqⁿ (2). So, for a good conducting medium

$$\nabla^2 E = \sigma \mu \frac{\partial E}{\partial t} \quad \text{---(11)}$$

and the attenuated solⁿ of this eqⁿ is

$$E = E_0 e^{-\beta r} e^{i(\alpha r - \omega t)} \quad \text{---(12)}$$

For a good conductor, if the frequency is not too high $\frac{\sigma}{\varepsilon\omega} \gg 1$

$$\alpha = \beta = \sqrt{\frac{\omega\sigma\mu}{2}} = \frac{1}{\delta}, \quad \text{where } \delta = \sqrt{\frac{2}{\omega\sigma\mu}} \quad \text{---(13)}$$

$$\Rightarrow E = E_0 e^{-r/\delta} e^{i(r/\delta - \omega t)} \quad \text{---(14)}$$

If $r = \delta$, the amplitude decreases in magnitude to $\frac{1}{e}$ times its value at the surface. So δ is a measure of the distance of penetration of an electromagnetic wave into a good conductor. The distance δ is called the skin depth. From eqⁿ (13) the ~~[illegible]~~ skin depth goes to zero as the conductivity approaches infinity and is small for good conductors at high frequency currents. Thus for copper $\sigma = 58 \times 10^6$ mho/m The skin depth at various frequencies is

60 Hz	8.5×10^{-3} m
1 MHz	6.6×10^{-5} m
30 GHz	3.8×10^{-7} m

Magnitude of skin depths ~~[illegible]~~ is different types of materials

Silver	100 MHz	10^{-7} m
Al	50 Hz	1.25×10^{-2} m
Sea water	30 kHz	10^{-1} m